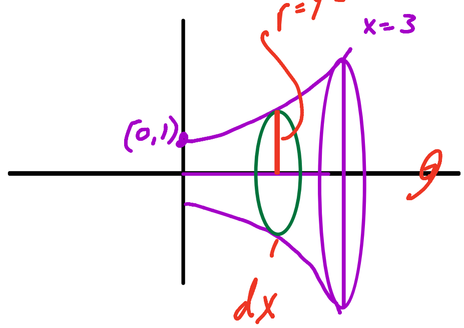
Period 1, March 10, 2025

1.



$y=e^x$, x-axis, $x=3$

$$\int_0^3 \pi r^2\,dx = \int_0^3 \pi y^2\,dx = \int_0^3 \pi (e^x)^2\,dx$$

$$\pi\int_0^3 e^{2x}\,dx = \pi\int e^u \cdot \frac{du}{2} = \frac{\pi}{2}\int e^u\,du$$

$u=2x$

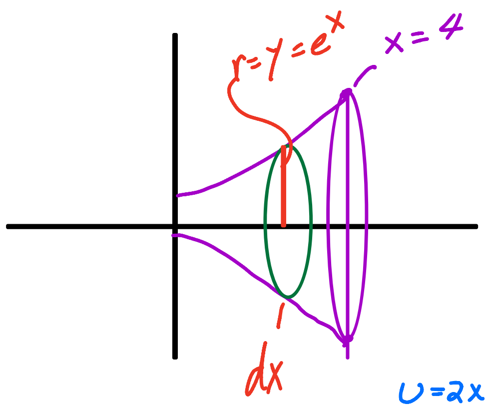
$du=2dx$

$\frac{du}{2}=dx$

$$\frac{\pi}{2}e^u + C = \frac{\pi}{2}e^{2x}\Big]_0^3$$

$$\frac{\pi}{2}\cdot e^{2\cdot 3} - \frac{\pi}{2}\cdot e^{2\cdot 0} = \frac{\pi}{2}(e^6 - e^0) = \frac{\pi}{2}(e^6-1)$$

$y=e^x$, x axis, $x=4$ around X-axis



$$\int_0^4 \pi r^2\,dx = \int_0^4 \pi y^2\,dx = \int_0^4 \pi(e^x)^2\,dx$$

$$\pi\int_0^4 e^{2x}\,dx = \pi\int e^u \cdot \frac{du}{2} = \frac{\pi}{2}\int e^u\,du$$

$u=2x$

$du=2dx$

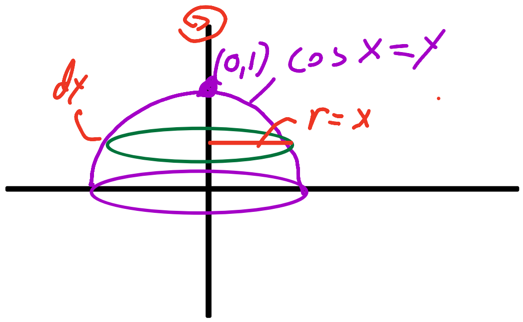
$\frac{du}{2}=dx$

$$\frac{\pi}{2}e^u + C = \frac{\pi}{2}e^{2x}\Big]_0^4$$

$$\frac{\pi}{2}e^{2\cdot 4} + \frac{\pi}{2}\cdot e^{2\cdot 0} = \frac{\pi}{2}(e^8 - e^0)$$

$$\frac{\pi}{2}(e^8-1)$$

I Quad, $y = \cos x$ around y-axis



$$\int_0^1 \pi r^2 \, dy$$

$$\int_0^1 \pi x^2 \, dy = \int_0^1 \pi (\arccos y)^2 \, dy = 3.5864$$

or

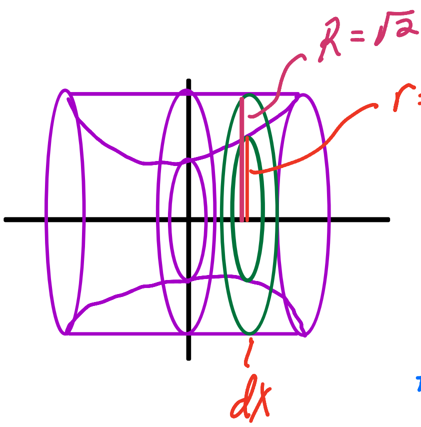
1.14159π

$\pi(\pi - 2)$

$y = \cos x$

$\arccos y = \arccos(\cos x)$

$\arccos y = x$



$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \pi (R^2 - r^2) \, dx$$

$$\pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} ((\sqrt{2})^2 - (\sec x)^2) \, dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (2 - \sec^2 x) \, dx$$

$$2x - \tan x \Big[_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

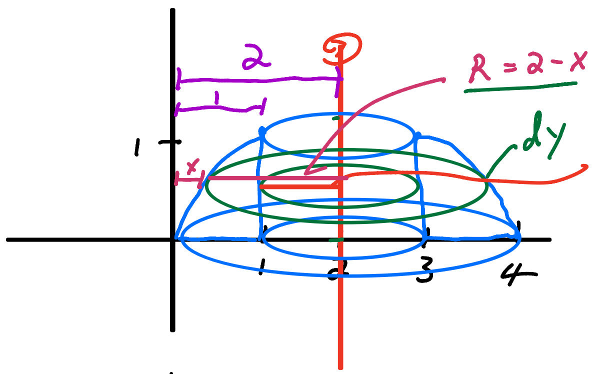
$-(-1)$

$$\pi\left(2\left(\frac{\pi}{4}\right) - \tan\frac{\pi}{4}\right) - \left(2\left(-\frac{\pi}{4}\right) - \tan -\frac{\pi}{4}\right) = \pi\left[\frac{\pi}{2} - 1 + \frac{\pi}{2} - 1\right]$$

$$\pi\left[\frac{2\pi}{2} - 2\right] = \pi(\pi - 2)$$

$y=\sqrt[3]{x}$, x-axis, $x=1$ around $x=2$

$y^3=x$



$$\pi\int_a^b (R^2-r^2)\,dy$$

$$(2-y^3)^2=(2-y^3)(2-y^3)$$

$$4-2y^3-2y^3+y^6$$

$$4-4y^3+y^6$$

$$\int_0^1 \pi(R^2-r^2)\,dy$$

$$\pi\int_0^1 (R^2-r^2)\,dy$$

$$\pi\int_0^1 [(2-x)^2-(1)^2]\,dy$$

$$\pi\int_0^1 [(2-y^3)^2-1]\,dy$$

$$\pi\int_0^1 [4-4y^3+y^6-1]\,dy$$

$$\pi\int_0^1 [y^6-4y^3+3]\,dy$$

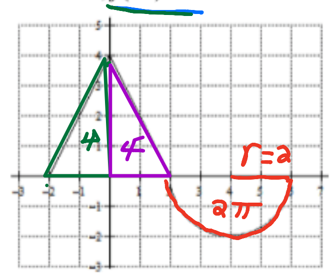
$$\pi\left[\tfrac{1}{7}y^7-y^4+3y\right]_0^1$$

$$\pi\left[\tfrac{1}{7}(1)^7-(1)^4+3(1)\right]-\pi\left[\cancel{\tfrac{1}{7}(0)^7-0^4+3\cdot 0}\right]$$

$$\pi\left[\tfrac{1}{7}-1+3\right]=\pi\left[\tfrac{1}{7}+2\right]=2\tfrac{1}{7}\pi=\tfrac{15}{7}\pi$$

The graph of $f'(x)$, the derivative of a function, $f(x)$, is pictured below on the interval [–2, 6]. Write and find the value of a definite integral to find each of the indicated values of $f(x)$ below. Also, $f(-2) = 5$.

41. Find the value of $f(0)$.	42. Find the value of $f(6)$.
$\int_{-2}^{0} F(x)' dx$ Area under the curve $F(-2) + 4$ $5 + 4 = 9$	$\int_{-2}^{6} F'(x)\,dx$ $5 + 4 + 4 - 2\pi$ $13 - 2\pi$

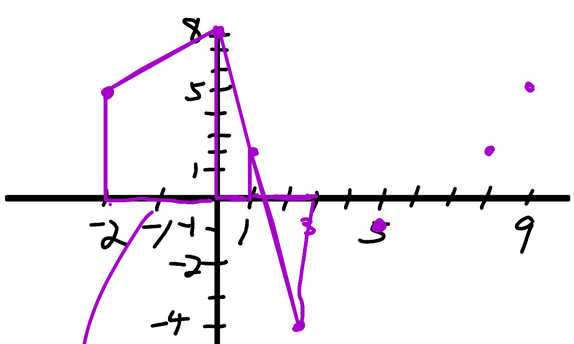


Graph of f'

23. Given the table to the right, approximate $\int_{-2}^{9} P(x)dx$ using six subintervals and a Trapezoidal sum.

2 1 2 2 3 1 Δx

x	–2	0	1	3	5	8	9
$P(x)$	5	8	2	–4	–1	2	5



$\frac{1}{2}(b_1 + b_2)h$

$\frac{1}{2}(5+8)\cdot 2 + \frac{1}{2}(8+2)\cdot 1 + \frac{1}{2}(2+-4)\cdot 2 + \frac{1}{2}(-4+-1)\cdot 2 + \frac{1}{2}(-1+2)\cdot 3 + \frac{1}{2}(2+5)1$

$\frac{1}{2}[13\cdot 2 + 10\cdot 1 + -2\cdot 2 + -5\cdot 2 + 1\cdot 3 + 7\cdot 1] = \frac{1}{2}[26 + \cancel{10} - 4 \cancel{-10} + 3 + 7]$

$\frac{1}{2}\cdot 32 = 16$

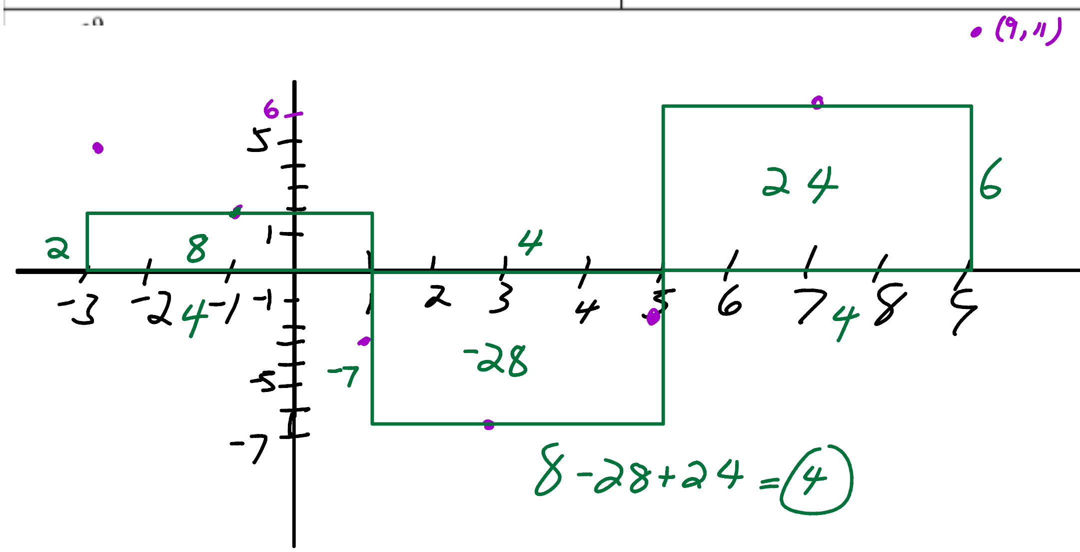
Given below is a table of function values of $h(x)$. Approximate each of the following definite integrals using the indicated Riemann or Trapezoidal sum, using the indicated subintervals of equal length.

4 +4 +4

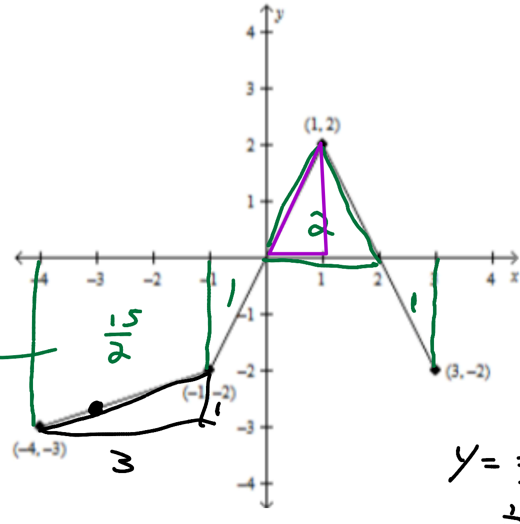
x	-3	-1	1	3	5	7	9
$h(x)$	5	2	-3	-7	-2	6	11

17. $\int_{-3}^{9} h(x)dx$ using three subintervals and a Midpoint Riemann sum.

18. $\int_{-3}^{3} h(x)dx$ using three subintervals and a Trapezoidal sum.



Pictured to the right is the graph of a function f. In exercises 30 – 35, find the values of each of the following definite integrals.



$\frac{1}{2}(-3+-2)\cdot 3$

$-\frac{15}{2}$

$m = \frac{1}{3}$

$y = \frac{1}{3}x + b$

$-3 = \frac{1}{3}(-4) + b$

$-1\frac{2}{3} = b$

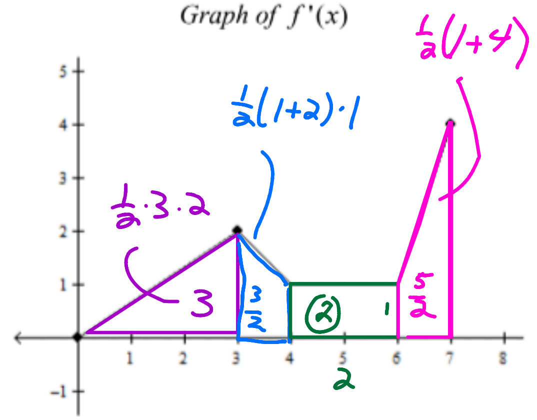
$y = \frac{1}{3}(x) - 1\frac{2}{3}$

$\frac{1}{3}(-3) - 1\frac{2}{3} = -1 - 1\frac{2}{3} = -2\frac{2}{3}$

30. $\int_{-4}^{2} f(x)dx = -\frac{15}{2} + -1 + 2$ $-7\frac{1}{2} - 1 + 2$ $-6\frac{1}{2}$	31. $\int_{0}^{3} f(x)dx = 2 + -1$ $= 1$	32. $\int_{-1}^{1} f(x)dx = -1 + 1 = 0$
33. $\int_{-4}^{0} f'(x)dx = F(0) - F(-4)$ $0 - {}^{-}3 = +3$	34. $\int_{-1}^{1} f'(x)dx = F(1) - F(-1)$ $2 - (-2)$ 4	35. $\int_{-3}^{2} f'(x)dx = F(2) - F(-3)$ $0 - (-2\frac{2}{3})$ $\cancel{7\frac{2}{3}}$ $2\frac{2}{3}$

Pictured below is the graph of $f'(x)$, the first derivative of a function $f(x)$. Use the graph to answer the following questions 38 – 40.

Graph of $f'(x)$



38. What is the value of $\int_0^7 f'(x)dx$ $= 3+\frac{3}{2}+2+\frac{5}{2}$

$5+\frac{8}{2}$

$5+4=9$

39. If $f(0) = -3$, what is the value of $f(3)$?

$-3+\int_0^3 F'(x)\,dx$

$-3+3=0$

40. If $f(3) = -1$, what is the value of $f(7)$?

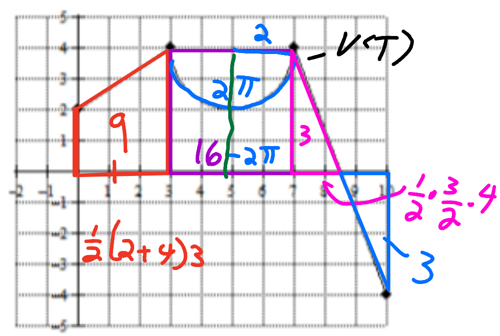
$-1+\int_3^7 F'(x)dx = -1+\frac{3}{2}+2+\frac{5}{2}$

$-1+2+\frac{8}{2}$

$-1+2+4=5$

The graph to the right represents the velocity, $v(t)$ in meters per second, of a particle that is moving along the x – axis on the time interval $0 \le t \le 10$. The initial position of the particle at time $t = 0$ is 12.

$T = 0 \quad S(T) = 12$



43. On what interval(s) of time is the particle moving to the left and to the right? Justify your answer.

NOT POSITION

44. What is the total distance that the particle has traveled on the time interval $0 \le t \le 7$. Leave your answer in terms of π. Indicate units of measure.

$$\int_0^7 |v(T)|\,dT = 9 + 16 - 2\pi = 25 - 2\pi$$

45. What is the net distance that the particle travels on the interval $5 \le t \le 10$? Round your answer to the nearest thousandth. Indicate units of measure.

displacement

$8 - \pi + 3 + -3$

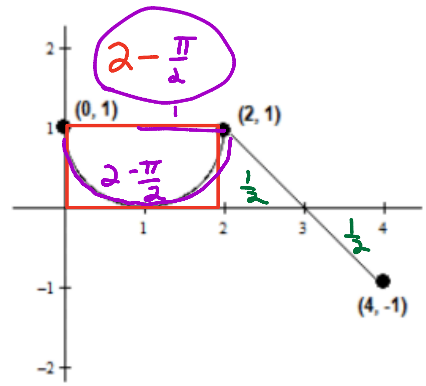
$8 - \pi$

47. What is the position of the particle at time $t = 5$? Indicate units of measure.

$9 + 8 - \pi$

$$12 + \int_0^5 v(T)\,dT = 12 + \underbrace{9 + 8 - \pi} = 29 - \pi$$

Pictured to the right is the graph of a function which represents a particle's velocity on the interval [0, 4]. Answer the following questions.



48. For what values is the particle moving to the right? Justify your answer.

$(0, 3)$

49. For what values is the particle moving to the left? Justify your answer.

52. What is the net distance that the particle travels on the interval [0, 4]?

$2-\frac{\pi}{2}+\frac{1}{2}-\frac{1}{2}=2-\frac{\pi}{2}$ Right

$\int |v(t)|\,dt$

53. What is the total distance that the particle travels on the interval [0, 4]?

$2-\frac{\pi}{2}+\frac{1}{2}+\frac{1}{2}=3-\frac{\pi}{2}$

Given below is a table of function values of $h(x)$. Approximate each of the following definite integrals using the indicated Riemann or Trapezoidal sum, using the indicated subintervals of equal length.

x	-3	-1	1	3	5	7	9
$h(x)$	5	2	-3	-7	-2	6	11

19. $\int_{-3}^{9} h(x)dx$ using six subintervals and a Trapezoidal sum.

$$\int_0^7 \left(5(\sin x)^2 + 3\sqrt{x}\right)dx$$

$= 53.3022591603$

$$\int_0^7 \left(2e^{(\sin t)^2} + 2\right)dt$$

$= 37.7218635081$

At time $t = 0$, there are 120 pounds of sand in a conical tank. Sand is being added to the tank at the rate of $S(t) = 2e^{\sin^2 t} + 2$ pounds per hour. Sand from the tank is used at a rate of $R(t) = 5\sin^2 t + 3\sqrt{t}$ per hour. The tank can hold a maximum of 200 pounds of sand.

59. Find the value of $\int_0^4 S(t)dt$. Using correct units, what does this value represent?

$\int (2e^{\sin^2 T} + 2)dT = 21.173 \rightarrow$ Pounds

60. Find the value of $\int_1^3 R(t)dt$. Using correct units, what does this value represent?

$$\int_1^3 \left(5(\sin x)^2 + 3\sqrt{x}\right)dx$$

$= 14.8781960017$ Pounds

61. Find the value of $\frac{1}{4}\int_0^4 S(t)dt$. Using correct units, what does this value represent?

62. Write a function, $A(t)$, containing an integral expression that represents the amount of sand in the tank at any given time, t.

63. How many pounds of sand are in the tank at time $t = 7$?

$120 + \int_0^7 S(T)dT - \int_0^7 R(T)dT = 120 + 37.72 - 53.302 = 104.418$

64. After time $t = 7$, sand is not used any more. Sand is, however, added until the tank is full. If k represents the value of t at which the tank is at maximum capacity, write, but do not solve, an equation using an integral expression to find how many hours it will take before the tank is completely full of sand.

$200 - 104.418 = 95.582$

$95.582 = \int_7^a S(T)dT$

A car travels on a straight track. During the time interval $0 \leq t \leq 60$ seconds, the car's velocity, v, measured in feet per second, and acceleration, a, measured in feet per second per second, are continuous functions. The table below shows selected values of these functions.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

54. Using appropriate units, explain the meaning of $\int_0^{60} |v(t)|dt$ in terms of the car's motion. Approximate this integral using a midpoint approximation with three subintervals as determined by the table.

55. Using appropriate units, explain the meaning of $\int_{15}^{50} a(t)dt$ in terms of the car's motion. Find the exact value of the integral.

56. Is there a value of t such that $a'(t) = 0$? If so, on what interval does such a value exist? Justify your reasoning.

57. Using appropriate units, approximate the value of $v'(31)$. What does this value say about the motion of the car at $t = 31$.